

Physical stability of the QED vacuum

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Abstract

The possibility of electron-positron pair creation by an electric field is studied by using only those methods in field theory the predictions of which are confirmed experimentally. These methods include the perturbative method of quantum electrodynamics and the bases of classical electrodynamics. Such an approach includes the back reaction. It is found that the vacuum is always stable, in the sense that pair creation, if occurs, cannot be interpreted as a decay of the vacuum, but rather as a decay of the source of the electric field or as a process similar to bremsstrahlung. It is also found that there is no pair creation in a static electric field, because it is inconsistent with energy conservation. The non-perturbative aspects arising from the Borel summation of a divergent perturbative expansion are discussed. It is argued that the conventional methods that predict pair creation in a classical background electric field cannot serve even as approximations. The analogy with the possibility of particle creation by a gravitational field is qualitatively discussed.

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1 Introduction

From the phenomenological point of view, one of the most important utilities of quantum field theory is the description of creation and destruction of particles in physical processes. On the other hand, from the theoretical point of view, the concept of particles is not a well-defined concept in quantum field theory. This is because the operator of the number of particles is not defined in terms of local quantum fields and their derivatives. For free fields, it is defined in terms of raising and lowering operators. This definition is

not unique because it depends on the choice of the complete orthogonal set of solutions to the free equations of motion. Although there exists a “natural” choice of solutions for free fields, the resulting definition of particles cannot be generalized uniquely when an interaction is present. Therefore, from the theoretical point of view, *all methods that describe creation and destruction of particles in physical processes are more or less vague*. From the pragmatic point of view, the theoretical problems with a method are not important if this method gives results that agree with experimental observations.

There exist several independent theoretical indications that strong electric fields cause electron-positron pair creation. However, this effect has never been measured and *the methods used to describe this effect have never been justified experimentally*. In [1], the imaginary part of the effective action of a quantum Dirac field interacting with a fixed classical electric field is found to be larger than zero, indicating that the absolute value of the vacuum-to-vacuum transition amplitude is smaller than 1. Although this gauge-independent description of the vacuum instability is often viewed as the clearest argument that a constant electric field causes pair creation, it has never been confirmed experimentally that the method used in [1] to calculate the *imaginary* part of the effective action correctly describes any physical effect. Pair creation by an electric field is also predicted by the Bogoliubov-transformation method [2, 3]. However, no physical effect resulting from a nontrivial Bogoliubov transformation has ever been observed. A tunneling picture can also describe vacuum instabilities, including pair creation by an electric field [4]. Again, the tunneling picture used to describe particle creation and destruction has never been justified experimentally. (Of course, it is experimentally confirmed that the tunneling picture may be applied to certain nonrelativistic quantum effects in which the number of particles does not change.)

Actually, the only experimentally justified method in quantum field theory that describes creation and destruction of particles is the perturbative method described by Feynman diagrams. Therefore, the most reliable approach to the study of the possibility of pair creation by an electric field seems to be this method. In Sec. 2 we use Feynman diagrams to study the possibility of pair creation in the field of a pointlike source. The possibility of pair creation by an electric field of a macroscopic source is discussed in Sec. 3. In Sec. 4 we prove that the QED vacuum is stable in a static classical background electric field and argue that the methods that predict an unstable vacuum in a classical background electric field cannot serve even as approximations. In Sec. 5 we discuss the non-perturbative aspects arising from the Borel summation of the divergent perturbative expansion and criticize the usual formal procedure that predicts an unstable vacuum by transforming an infinite real quantity into a finite complex quantity. The analogy with the possibility of particle creation by gravitational fields is qualitatively discussed in Sec. 6, while the conclusions are drawn in Sec. 7.

2 The description in terms of Feynman diagrams

Our basic philosophy is to use only those methods the predictions of which are confirmed experimentally. Pair creation by an electric field, as well as any kind of vacuum instabilities, has never been observed. Therefore, in this section we base our analysis on

an analogy with the observed effects the theoretical description of which is similar to the corresponding description of pair creation.

Let us start with the discussion of elastic electron scattering in a classical background electric field. At the first order of perturbation theory, it is described by the diagram in Fig. 1(a). We assume that the electric field is static and vanishes at infinity, which implies that the initial energy of the electron at infinity is equal to the final energy of the electron at infinity. However, since the presence of the electric field breaks the translational symmetry, the initial 3-momentum \mathbf{p}_i does not need to be equal to the final 3-momentum \mathbf{p}_f .

Of course, the 3-momentum of the whole system must be conserved. The picture in which the electric field is a fixed classical background is only an approximation. First, the change of the electron 3-momentum causes a back reaction on the electric field. Second, the electromagnetic field should also be quantized. As far as the initial and final asymptotic states are all that interests us, a consistent description with a quantized electromagnetic field and the back reaction included is described by the diagram in Fig. 1(b). It is manifest in this diagram that the 3-momentum is conserved because the charged particle represented by the double line recoils. A fixed classical electromagnetic field is now replaced by a “virtual photon”, i.e., by the propagator of the electromagnetic field

$$D_{\mu\nu}(q) = -\frac{1}{q^2} \left(g_{\mu\nu} + (a-1) \frac{q_\mu q_\nu}{q^2} \right). \quad (1)$$

The parameter a parametrizes the class of gauges that satisfy the Lorentz condition $\partial_\mu A^\mu = 0$. Although we do not know a general explicit proof that the physical effects of the electromagnetic-field quantization do not depend on the choice of gauge, it is well known that these effects do not depend on the choice of a , i.e., on the choice of gauge that satisfies the Lorentz condition. Besides, *the phenomenological predictions of QED based on the propagator (1) agree with experimental observations.*

If the mass M of the particle represented by the double line in Fig. 1(b) is much larger than the electron mass m (for example, this heavy particle may be a proton treated as an elementary particle or a muon), then a static approximation is reasonable. In this approximation, both the initial and the final 3-momentum of the heavy particle are equal to zero. Consequently, in the Feynman gauge $a = 1$, the effect of the propagator (1) is the same as that of the fixed classical electromagnetic background described (in the momentum space) by the electromagnetic potential $A^i = 0$, $A^0 \propto \mathbf{q}^{-2}$. This corresponds to a static electric field caused by a pointlike charged source. In this way, the approximation corresponding to Fig. 1(a) is derived from an “exact” result (at the first order of perturbation theory) represented by Fig. 1(b). Note that the calculation of the diagram in Fig. 1(a) depends on the choice of gauge that satisfies the Lorentz condition and describes a given electric field. Therefore, the “exact” diagram in Fig. 1(b) tells us that the “correct” gauge in Fig. 1(a) describing a static electric field is

$$A^\mu = (A^0(\mathbf{x}), 0, 0, 0). \quad (2)$$

The physics of an electron scattering in the field of a pointlike massive charge can be understood even with classical physics. Since pair creation is a genuine quantum effect,

it is instructive to discuss the $\gamma \rightarrow e^+e^-$ process in the electric field of a nucleus, as an example of a *measured* [5] genuine quantum effect, similar to the e^+e^- creation by an electric field. It is described by the diagram in Fig. 2(a). Assuming again that the electric field is static and vanishes at infinity, we see that the energy is conserved again, while the 3-momentum is not. The “exact” diagram, in which the 3-momentum is also conserved, is shown in Fig. 2(b). Similarly to the case of electron scattering, we see that the final effect of the back reaction is a recoil of the heavy source of the electric field.

We are now ready to discuss the possibility of pair creation by an electric field. It is described by the diagram in Fig. 3(a). From the kinematics, it is easy to see that such a pair creation may be consistent with the 3-momentum conservation, but cannot be consistent with the energy conservation. On the other hand, if the electric field is static and vanishes at infinity, then energy must be conserved. In other words, the pair cannot be created, simply because the kinematics forbids it. This fact is even clearer from the “exact” diagram in Fig. 3(b).

Actually, there is a way to make the diagram in Fig. 3(b) consistent with the 4-momentum conservation, provided that the final heavy mass M_f is not equal to the initial heavy mass M_i . (The usual QED Lagrangian does not contain such a vertex, but an effective Lagrangian for a composite particle such as a nucleus could contain it.) Obviously, the process is possible if $M_i \geq M_f + 2m$. In that sense, pair creation is possible. However, it does not mean that the vacuum may be unstable. Since the mass of the source of the “classical background” changes during the process, this process is naturally interpreted as a spontaneous decay of a particle with the mass M_i into the e^+e^- pair and a particle with the mass M_f . If the initial heavy particle is stable, then the electric field of this particle does not create e^+e^- pairs.

Another way of making the diagram in Fig. 3(b) consistent with the 4-momentum conservation is to assume that the double-line particle is accelerated by some external force. The process is similar to bremsstrahlung, in which outgoing photons are replaced by outgoing electron-positron pairs. In this case, the energy needed for pair creation comes from the agency that accelerates the double-line particle.

3 Pair creation by a macroscopic electric field

In the preceding section we have seen that the back reaction is automatically included when the electromagnetic field and its source are quantized. We have also seen that all effects of this back reaction are easily understood by classical kinematics related to the 4-momentum conservation. On the other hand, when the source of the electromagnetic field is not one particle, but a huge number of particles, then it is not easy to describe the electromagnetic field and its source by a quantum formalism. However, the interactions among particles are essentially the same, so, again, one can apply the classical conservation laws in order to determine whether pair creation is possible. The only significant difference is the fact that particles that constitute the macroscopic source may not be separated in the final and initial state, so one cannot say that the initial and final states of particles are free states. Consequently, not all consequences of perturbative techniques can be applied. However, the basic understanding of classical electrody-

ics is sufficient to give a qualitative description of the macroscopic electromagnetic field and its source.

Consider, for example, the possibility of pair creation by an electric field produced by a capacitor consisting of two large, parallel, and oppositely charged conducting plates. Such a configuration may be relevant to an experimental investigation of pair creation. Since the charge of the plates is fixed, the electric field between the plates depends only on the relative position of the plates. Since this field may be approximated by a constant electric field between the plates, one could expect that the pairs are created [1, 2, 3, 4] with a thermal distribution in energy [6, 7, 8]. However, the pair creation cannot be consistent with the conservation of energy. If the pair is detected far away from the capacitor, then the presence of the pair does not influence the energy of the plates and the energy of the electric field between them. Therefore, the back reaction cannot save the conservation of energy, implying that the pair is not created.

By changing some experimental conditions, there is still a possibility for pair creation without a decay of particles that constitute the plates. Since the pair creation increases the energy of the system, the creation should be accompanied by an decrease of the capacitor energy. The capacitor energy will decrease if the plates of the capacitor come closer. However, if the position of the plates is fixed (by some strong mechanical force), then the plates cannot come closer and there will be no pair creation. On the other hand, if the plates are not fixed, then the electric field will attract them, converting the energy of the electric field into the kinetic energy of the plates. If pair creation also occurs during this collapse of the plates, then the conservation of energy implies that the increase of the kinetic energy of the plates will be smaller than the decrease of the field energy. Therefore, on the macroscopic level, *pair creation is a dissipative process that manifests as a friction force* that acts on the plates. This supports the similarity with bremsstrahlung, which is also a dissipative process that acts as a friction.

The pair creation by a capacitor with unfixed plates can also be interpreted as a decay. In this interpretation, it is the capacitor itself that decays into a lower-energy state, because the initial state has a large potential energy stored in the electric field, while the final state, in which the plates are close to each other, has much lower potential energy. Part of this energy is transmitted into the produced pairs.

4 Vacuum-to-vacuum transition amplitude

In the preceding sections we have seen that pair creation in an electric field is possible under certain circumstances. If the source of the field is a pointlike particle, then the probability and distribution of the created pairs can be calculated by well-understood and experimentally justified methods described by Feynman diagrams. Quantum electrodynamics *without* an external classical electromagnetic background is sufficient, but the description based on a classical background may serve as an approximation.

If the source of the electric field consists of a huge number of particles, then the calculation of Feynman diagrams is not an efficient method for calculating the effect. Our discussion does not suggest how to calculate it in this case. One could guess that the conventional methods [1]-[4] that describe pair creation by classical electric fields

are good approximations. Although they are not consistent with the conservation of energy, one could assume that these methods may serve as an adiabatic approximation, because the pair creation is slow, so the violation of the energy-conservation law is small. Assuming this, one can modify the method by introducing the effects of back reaction [9]-[13]. In this section, we argue that the methods [1]-[4] are completely wrong, i.e., they cannot serve even as approximations.

Take, for example, an approximation in which the Dirac field is quantized and interacts with a fixed static electric field. The method presented in [1] predicts that the absolute value of the vacuum-to-vacuum transition amplitude is smaller than one, suggesting an unstable vacuum. On the other hand, the method based on Feynman diagrams shows that the vacuum cannot decay simply because the amplitudes $\langle n, \text{out} | 0, \text{in} \rangle \equiv \langle n | S | 0 \rangle$ vanish for all physical on-shell states $|n\rangle$ orthogonal to the vacuum $|0\rangle$. The unitarity implies that the Feynman-diagrams approach must give

$$|\langle 0 | S | 0 \rangle| = 1 , \quad (3)$$

in contradiction with the result of [1]. Let us show explicitly that (3) is the correct result.

The Lagrangian of the system is

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} , \quad (4)$$

where

$$\begin{aligned} \mathcal{L}_{\text{free}} &= \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi , \\ \mathcal{L}_{\text{int}} &= -e\bar{\psi}\gamma_\mu\psi A^\mu . \end{aligned} \quad (5)$$

The background A^μ is of the form (2). Since we use the interaction picture, the quantum Dirac field satisfies the free Dirac equation, so it can be expanded as

$$\begin{aligned} \psi(x) &= \sum_s \int \frac{d^3k}{(2\pi)^{3/2}} \sqrt{\frac{m}{\omega(k)}} \\ &\times [b_s(k)u_s(k)e^{-ikx} + d_s^\dagger(k)v_s(k)e^{ikx}] , \end{aligned} \quad (6)$$

where $u_s(k)$ and $v_s(k)$ are free spinors [14], $b_s(k)$ destroys electrons, and $d_s^\dagger(k)$ creates positrons. The S-matrix is given by

$$S = T e^{i \int d^4x \mathcal{L}_{\text{int}}} . \quad (7)$$

Therefore, the exponent in (7) is proportional to

$$\int d^3x A^0(\mathbf{x}) \int_{-\infty}^{\infty} dt \bar{\psi}(\mathbf{x}, t) \gamma_0 \psi(\mathbf{x}, t) . \quad (8)$$

When the expansion (6) is used in (8), then four types of terms appear, i.e., the terms proportional to $b_s^\dagger(k)b_{s'}(k')$, $d_s(k)d_{s'}^\dagger(k')$, $b_s^\dagger(k)d_{s'}^\dagger(k')$, or $d_s(k)b_{s'}(k')$. (The time-ordering changes the ordering of the operators, which we ignore because it does not influence our

conclusions.) This implies that $S|0\rangle$ is a fermion variant [15, 16] of a squeezed state [17, 18]. The second two types of terms are responsible for the squeezing, i.e., for the nontrivial particle content of the state $S|0\rangle$. If these terms are missing, then the first two types of terms only change the phase of the vacuum.

The time integration can be performed before the 3-momentum integrations. Therefore, the time integration of the first two types of terms leads to a factor proportional to $\delta(\omega - \omega')$, while that of the second two types of terms leads to a factor proportional to $\delta(\omega + \omega')$. Since $\omega + \omega'$ cannot be zero, the integration over the 3-momenta kills the second two types of terms. Therefore, only the first two types of terms appear in the final expression, which implies that the S -matrix operator does not change the number of particles when acts on a state with a definite number of particles. The function $\delta(\omega - \omega')$ provides that the energy is conserved in any process described by the S -matrix element $\langle f|S|i\rangle$. When S acts on the vacuum, it merely changes its phase. This proves Eq. (3), i.e., the stability of the vacuum in a static electric field. It also clearly shows that this stability is directly related to the conservation of energy, i.e., to the factor $\delta(\omega + \omega')$ which appears because A^μ does not depend on time.

Recall that the method used above to prove (3) is justified by experiments, while there is no such justification for the method used in [1] to show that (3) is not true. It is also important to stress that, although the method in [1] is very different from the method used above, the *physical* assumptions and approximations are the same. Therefore, one of the *methods* must be completely wrong; different results cannot be manifestations of different physical assumptions and approximations. All remarks made in this paper clearly indicate that it is the method in [1] that must be wrong. Actually, it has already been shown by an independent argument that the method presented in [1] is inconsistent [19].

Our discussion suggests that other methods [2, 3, 4] that predict pair creation by a classical static background electric field should also be wrong. The inconsistencies of these methods have also been discussed by independent arguments [19, 20].

5 Non-perturbative aspects

One could argue that the result of Sec. 4 differs from that of [1] (for a constant electric field) because the latter is a non-perturbative result, while the former is based on the perturbative expansion. (By a perturbative result we mean a mathematical expression written as a series expansion in e without negative powers of e .) In other words, one could object that the calculation in Sec. 4 is wrong because the non-perturbative contributions are not included in this calculation. However, as shown in [21], *the non-perturbative result of [1] does not arise from a contribution which is not present in a perturbative expansion*. The calculation in [21] is not based on the interaction picture, but, as we demonstrate below, essentially the same arguments can be applied to the calculation based on the interaction picture of Sec. 4.

By calculating all contributions to $\langle 0|S|0\rangle$ for A^μ of the form (2), we obtain

$$\langle 0|S|0\rangle = \exp[iW(e)] , \quad (9)$$

where $W(e)$ is given by the sum of all different one-loop diagrams without external legs. It can be written as

$$W(e) = \sum_{n=0}^{\infty} c_n e^n , \quad (10)$$

where c_n are functionals of $A^0(\mathbf{x})$. They are real because $A^0(\mathbf{x})$ is real. To understand how a non-perturbative result may arise from a perturbative result (10), it is not necessary to calculate c_n explicitly. It is crucial to know that the series (10) is divergent, even if the coefficients c_n are made finite by some regularization or renormalization procedure. Assume, for example, that they are of the form [21]

$$c_n = \alpha^n n! , \quad (11)$$

where α is some real positive constant. Clearly, the series (10) with coefficients (11) has a zero radius of convergence. Since the coefficients do not alternate in sign, even the Borel summation does not give a convergent result. Nevertheless, let us do the formal Borel summation. By writing

$$n! = \int_0^{\infty} ds s^n \exp(-s) , \quad (12)$$

putting this in (11), and interchanging the order of summation and integration in (10), we obtain

$$W(e) = \frac{1}{\alpha e} \int_0^{\infty} dz \frac{1}{1-z} \exp\left(-\frac{z}{\alpha e}\right) , \quad (13)$$

where a new integration variable is introduced:

$$z = \alpha e s . \quad (14)$$

If (13) is expanded in powers of e and (14) is ignored, then negative powers of e appear. In other words, (13) is a *non-perturbative* result. We also see that the formal Borel summation given by (13) did not change the fact that $W(e)$ is *real*, corresponding to a stable vacuum in (9).

One could be worried by the fact that (13) is *divergent*, which is a consequence of the pole of the subintegral function at $z = 1$. Therefore, one could avoid the pole by deforming the contour of integration [21], which leads to a finite real part of $W(e)$ and a positive *imaginary* part of $W(e)$. If such a formal procedure of giving an imaginary part to a real quantity makes sense, then it implies that (13) corresponds to an *unstable* vacuum. However, it has already been shown that such an artificial formal procedure is not consistent [19]. Below we give new physical arguments against such a deformation of the integration contour.

First, the fact that $W(e)$ is divergent is not a problem. The physical quantity calculated from (9) is $|\langle 0|S|0\rangle|^2$. It is equal to 1 for any real number W , including the case in which W approaches infinity along the real axis. Therefore, there is no physical reason to deform the integration contour. Moreover, the fact that (13) is divergent does not imply that it is infinite. Actually, the infinite contribution from $z = 1 - \epsilon$ (with $\epsilon \rightarrow 0^+$) is canceled by the infinite contribution from $z = 1 + \epsilon$. In other words, the

principal value of the integral in (13) is *real* and *finite*. The same is true for similar divergent integrals that appear in [1] and [21] related to the explicit calculation of the vacuum-to-vacuum transition amplitude in a constant background electric field. This finite and real principal value is the most natural choice for these divergent integrals, which leads to a stable vacuum.

Second, assume that the contour should be deformed, so that the vacuum is unstable. As we have seen, although (13) is a non-perturbative result, all contributions to it arise from the Feynman diagrams. If the vacuum instability realizes as a pair creation, then there must be a non-zero probability of producing any particular number of pairs. For example, the probability of producing one pair is given by the diagram in Fig. 3(a). However, there is no a consistent way of making this diagram non-vanishing for A^μ of the form (2). In particular, this would be in contradiction with the conservation of energy. The “need” for a non-zero value arising from the diagram in Fig. 3(a) does not emerge from any particular diagram contributing to (10), but from the whole diverging sum. Therefore, avoiding this divergence by an integration-contour deformation does not manifest as a modification of particular diagrams. In other words, it is not consistent to replace infinite real quantities by finite complex quantities in a way described above.

To summarize, the non-perturbative aspects are artefacts of Borel summation of a divergent series, while the vacuum “instability” is an artefact of an integration-contour deformation. The divergence of $W(e)$ does not cause any physical problems, while the integration-contour deformation is physically unjustified. Therefore, in our opinion, the physical interpretation of formal results in [1] and [21] related to vacuum instability in a constant background electric field is wrong.

6 Analogy with the gravitational particle creation

There is a lot of similarity between the particle creation by a classical electromagnetic background and that by a classical gravitational background [3, 4, 6, 8, 22]. Unfortunately, it is still not known how to quantize gravity consistently, so it is not clear if it would be consistent to base a quantitative analysis on a diagram analogous to the diagram in Fig. 3(b). Nevertheless, by analogy, we can use the final results of the preceding sections to draw some qualitative conclusions related to the gravitational particle creation.

The particle creation by a capacitor has many similarities to that by a black hole. First, since the electric field between the plates of the capacitor is approximately constant, one could expect a thermal distribution of the particles created [6, 7, 8], just as is expected for black holes [23, 24]. However, we have shown that a static capacitor does not create particles; only a collapsing capacitor can do it. Similarly, we expect that if some mechanism prevents the gravitational collapse, then the black hole does not create particles.

In analogy with the electromagnetic case, we may also speculate about the effect of the back reaction related to the particle creation by the source of the gravitational field. Our discussion suggests that the back reaction manifests as a friction on the moving matter, which slows down the gravitational collapse. If a final state of matter

(determined by the yet unknown physics on the Planck scale) is reached, then the collapse and the particle creation stops.

Another interesting question related to particle creation by gravitational fields, especially by black holes, is *where* particles are created. The analogy with the electromagnetic case suggests that the matter, i.e., the source of the gravitational field, is the source of the created particles. If this is true, then *the created particles that escape from the black hole are created by matter that has not yet fallen into the hole bounded by the horizon*. This picture is drastically different from the usual picture that describes the black-hole evaporation [23, 24]. Nevertheless, although the mechanism of particle creation by black holes suggested by our discussion is completely different from the usual one, we still expect a thermal distribution of escaped particles, as seen by a distant observer. This is because the thermal distribution is a consequence of the exponential red shift and can be understood even by classical physics [25, 26], without any assumption on the physical mechanism that causes particle creation near the horizon.

If the analogy with bremsstrahlung is used, then, since the matter moves inertially in the gravitational field, particle creation may not exist at all. Such a reasoning is reasonable if the equivalence principle is applicable on the level of quantum gravity.

As we see now, the predictions of the conventional methods that describe particle creation by classical gravitational backgrounds may be completely wrong. The inconsistencies of these methods have already been discussed by independent arguments [19, 20, 27]. A reliable prediction can be given only by a quantum-gravity theory that automatically includes the effects of back reaction.

7 Conclusions

Pair creation by an electromagnetic field can be consistently studied only if the electromagnetic field and its source are quantized. Using the experimentally verified method of Feynman diagrams, we have found that the pairs are not created if the source of the field is a stable unaccelerating particle. Similarly, if the source of the field is a macroscopic static object, then the pairs are not created either. In certain cases there is the possibility of pair creation, but such an effect should not be interpreted as a vacuum instability. Instead, such a process can be interpreted as a decay of the source of the electromagnetic field or as a process similar to the bremsstrahlung. Although these results may seem to be trivial consequences of perturbative QED and the energy-conservation law, these results clearly indicate that the methods that predict pair creation by classical background electromagnetic fields are inappropriate. They cannot serve even as approximations.

Analogous remarks are also valid for the particle creation by a gravitational field. In particular, this implies that our understanding of black-hole thermodynamics should be drastically revised.

Yet, an approximation in which quantum particles move in classical electromagnetic and gravitational backgrounds should certainly make sense in some cases. A consistent general-covariant and gauge-invariant approximation that describes the particle content of quantum fields in classical backgrounds will be given elsewhere.

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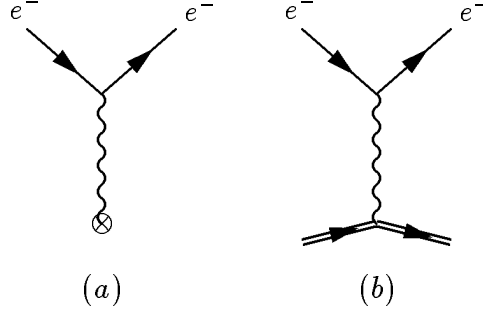


Figure 1: The lowest-order diagrams representing elastic electron scattering caused by the electromagnetic interaction. In diagram (a), the electromagnetic interaction is described by a fixed classical electromagnetic background. In diagram (b), the electromagnetic interaction is described by a quantum virtual photon. When the double-line particle is much heavier than the electron, then diagram (b) can be approximated by diagram (a) with a potential $A^\mu = (A^0(\mathbf{x}), 0, 0, 0)$ describing the electric field of a pointlike charge.

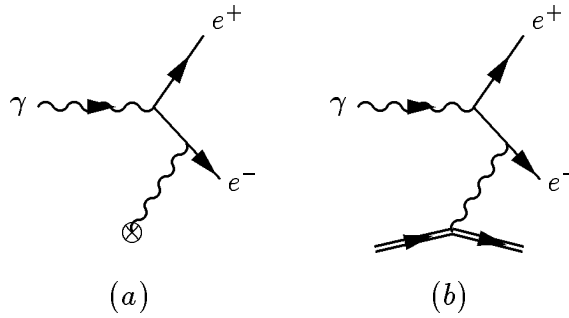


Figure 2: The lowest-order diagrams representing the $\gamma \rightarrow e^+e^-$ process in the field of a pointlike charge. (The diagrams in which the wiggly line without an arrow is connected with the upper fermion line are omitted.) As in Fig. 1, diagram (a) represents an approximation of diagram (b).

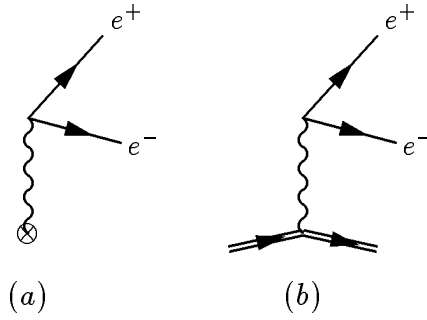


Figure 3: The lowest-order diagrams representing e^+e^- pair creation caused by the electromagnetic interaction. As in Fig. 1, diagram (a) represents an approximation of diagram (b). If the double line represents a particle for which the final mass is equal to the initial mass, and if an external force does not act on this particle, then, owing to energy conservation, the amplitude corresponding to diagram (b) vanishes.